14   FROM ZERO TO INFINITY

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Generally Zero is considered as if there is nothing. But nothing cannot be Zero. It has got its own meaning. It may be a space which can accomodate everything, all beings and things originated from this golden womb i.e. Hiranyagarbha. It is the symbol of that Great, who is greatest of all. Numbers are nine in total as Prakriti, Aadishakti -Nava Durga and Zero is one only - The Purusha - the Shaktiman. Although he is, everything but without shakti he is Zero. As soon as it joins with prakriti it grows into numbers. The visible world is the product of the union of purusha and prakriti. Medium of expression of this world and the whole universe is nothing but the numbers. Without number one cannot communicate the reality to anyone.

Perhaps, Professor Ramakrishna Rao is more erudent in his approach and inquiry and he is here to be read and appreciated.

Editor

1. Introduction

The concept of Zero and Infinity and their related subjects have been very interesting in the history of numbers and mathematics itself, as it is one of the most important technical invention made in the history of humanity .It is surprising to note that most of the so called advanced scientific nations of the world did not have such concept, till it was introduced into their workings. The inventors of zero, Infinity ,the numerals and the decimal system what we use today should have applied much thought to devising such notation and symbolism. Though, Hindus, have been credited with such invention the books and reference works coming from the western and other quarters have been throwing shadow of doubt on them. Therefore, it is important to study as to how the inventors were able to invent such a number system, which has been the bedrock of all scientific thinking and thought, from which the inventors could be identified scientifically and historically.

2. The Attitude of Westerners towards Zero and Infinity

The westerners themselves have faithfully recorded to accept that in the late 12th century only, the Europeans actually began to make use of the zero and decimal system, reportedly introduced to them through Arab traders bringing them from India. Though they were having idea about the numbers 1 to 9, they could not understand zero, as it affected considerably all of their thinking processes. Their concept of Numbers was only associated and imbibed with counting, calculation and quantification. Moreover, their calender had never any zero year, though, they claim that the calender start from B.C. They considered zero as an intellectual obstacle[2]. Without understanding the significance of it, they even opposed the use of it. The resistance took two forms. some considered

them as the creation of devil. while others made fun and ridiculed them[3]. As for Infinity, its usage was in 17th-18th centuries. John Wallis (1616-1703) is claimed to have introduced the Infinity symbol. Even after the introduction of Calculus in 17th century, the western mind was puzzled with infinity and infinitesimals. The puzzle persisted for nearly two centuries, unit Cauchy (1789-1857) and Weirstrass (1815-97) showed how the awkward notion could be eliminated[4].

3. Who Used Zero and Infinity First?

Western scholars reported that Babylonians and Mesomericans (Mayans, Incas etc.,) were using their own zeroes, but they were not fully developed in their conceptual and operational meaning in application. As the former used sexagesimal (60 as base) and latter vigesimal (20 as base) systems, and it was not designed for the normal mathamatical operations consonant with decimal system. Both systems did not contain zero symbol to represent zero. There is a claim that Muslims invented "Arabic numerals " even before Hindus, including zero[5].

According to Indian source, the earliest usage of zero is traced back to Pingala in his Chandha- sutra (c.200 BCE) and Panini in his Ashtadhyai (c.500 BCE)[6] But the philosiphical usage of it can be traced back to the Vedas[7]. The Vedic literature has been very specific that origin of everything has been "akasha" empty place, one of the words used to denote zero and "Hiranya garbha", the golden womb the place of origin of all life in the round shape, obviously to symbolize such concept of emptiness or the place of origin. As mathematical logic is closely connected with philosophy and thought processes, the philosophical aspect can be appreciated.

The Hindus have definitely conceived the idea of infinity equating with that of Brahman and Atman. There are many Vedic (3500 BCE) and Upanishadic (2500 BCE) references to prove the fact.

Mathamatically, Brahmagupta (628 BCE) was the first to attempt the division by zero and named such resultant quantity as Khacheda (destroyer of zero)

The first usage of infinity in mathematical operations is found in the Bija Ganita of Bhaskaracharya (1114-1185 CE). He mentioned it as 'khahar' (remover of zero). He mentions:

"Statement: Dividend 3, Divisor 0, Quotient the fraction 3/0. This fraction of which the denominator is cipher, is termed an infinite quantity...... In this quantity consisting of that which has cipher for its divisor, there is no alteration though many may be inserted or extracted; as no charge takes place in the infinite and immutable God, at the period of the destruction or creation of worlds, though numberous orders of beings are absorbed or put forth"[8],

implying that he knew that $n/0 = \infty$

Ganesa (1545CE) notes that n/o is:

".........an indefinite and unlimited or infinite quantity; since it cannot be determined how great it is. It is unaltered by the addition; or subtraction of finite quantities: since in the preliminary operation of reducing both fractional expressions to a common denominator, preparatory to talking their sum or difference, both numerator and denominator of the finite quantity vanish"[9].

Krishna has also recorded:

As much as the divisor is diminished, so much is the quotient increased. If the divisor is the utmost, the quotient is to the utmost increased. but, if it can be specified, that the amount of the quotient is so much, it has not been raised to the utmost: for a quantity greater than that can be assigned. The quotient, therefore, is indefinitely great, and is rightly termed infinite"[10].

Coming to the infinity the western sources as usual point to the Greeks, Particularly, Anaximander (c.611-545 BSE,) Leucippus (400 BSE) and zero of Elka (c.450 BSE) for the conception of infinity. But, their conception was superficial and they had not specifically mentioned anything about it. As they were tired of thinking about Infinity,"..... **the classical mathematicians banned from rigorous thought, the infinite in all its forms -the infinitesimal, the infinitely great, and infinite multiplicity**"[11]. They, "even avoided it, by their elegant method of exhaustion".

4. The Words used for zero and Infinity and their Significance

The zero is mentioned as sunyam, 'kha' and 'bindhu'[12] and also by the words gagana, viyat, akasha, ambara, abhra, anata, vyoma.

The word **sunya/sunyam** is derived from the root sun (p.p), which has the following meanings:

1. born, produced
2. blown,blossomed, opened, budded
3. empty, vacant

Sunyam has the following meanings:
1. bringing forth, paturation
2. a bud, blossom
3. a flower

Shunya as an adjective has the following meanings:
1. empty, void
2. Vacant (applied to heart, glances & c), absent listless
3. non- existence
4. lonely, desolate, secluded, deserted
5. dejected, downcast, de-spirited
6. utterly devoid of deprived of, without, wanting in
7. indifferent
8. guileless
9. non-sensical, unmeaning
10. bare, naked

Sunyam has the following meanings:

1. a vacuum, void, blank
2. the sky, space, atmosphere
3. a cipher, dot
4. non-entity, (absolute) non-existence

Thus, it is evident that the concept of zero is connected with origin of life, universe and number system it self.

The words used for Infinity are aanantyam, anantata, aparta, simabhav, asimata and parardhye.

Anantyam- 1. Infinity, endless(in time, space or number) as in" anantyadh.
2. Boundless
3. Immortality, eternity.

Anantata- 1. The earth (the endless)
2. the number one
3. noun of Parvati
4. noun for various plants like Sakhi, Anantamulam, Dhurva etc.

Anantam- 1. The sky, aymosphere
2. Infinity
3. Absolution
4. The Supreme Spirit, Brahma (Parabrahmam)

Aparta- 1. Shoreless
2. Boundless, unlimited
3. Inexhaustible, immense
4. out of reach
5. Difficult to be crossed, surmounted or overcome

Simabhav- The idea, nature or application of limits (i.e. the highest or utmost limit)

Asima - Endless, boundless, limits.

Monier Williams adds the following meanings to the word anata- 23rd lunar asterism and a periodic decimal fraction (?) implying its connection with mathematics. Thus, the words expressed denote

specifically the Infinity that cannot be incomprehensible, but understood with already conceptualized and known typical examples.

5. Are zero and Infinity Numbers?

Zero was not considered as a number in the western world and civilizations until modern times[13]. In fact they were not having any zero year crossing from BCE to CE i.e.,-5-,4-,3-,2-,1,0 1,2,3,4,5 and so on! But according to Indian system, zero is a number used as a value of a variable.

The Hindu mathematicians initially considered infinity as a number like zero, though modern mathematicians do not consider so. As the quantity of "limitless, boundless, very great" is limited mathematically, Infinity" is finitely defined. With the conceptual development, we can understand that infinity can only be approached, but not reached! The operations of infinity with zero and other numbers explained by Bhaskara II, Ganesha and Krishna clearly show that slowly, they too considered it as a mathematical entity, but not a number. Any number how much it might be great is finite. Only George Canter in his set theory shows that the existence of many infinities. He also discovered the *transinfite numbers*, which are related to the concept of Infinity. This again amounts to acceptance of many distinct infinities, but they are identified finitely.

6. Can Numbers Function without Zero and Infinity?

How mathematical problem can be solved or at first can be thought of without Zero and Infinity? Without zero, the formation and operation of numbers cannot be imagined. Mathematical thinking process could not go before and beyond 1 to imagine the existance of negative numbers. Square root of any number cannot be found. The concept of natural numbers, prime numbers and complex numbers cannot be conceived and understood without zero. Thus, the absence of zero or "zero knowledge of zero" hampers all mathematical processes. Therefore, it can be asserted that without zero, there is no mathematical and scientific thinking processes leading to any invention. That Aryabhata (479 CE) could calculate the value of $Pi = 3.414$ accurately proves the continuous usage of zero in mathematical operations.

What would happen, if mathematicians, physicists and other scientists do not have infinity for their operations? Can they imagine such intricate concepts, evolve hypothesis and formulate theories of Relativity, Contraction of Length, etc, in physics, Infinite series, Infinite Vector, Infinite Space, Axiom of Infinity, Set Theory, Continuum hypothesis in mathematics ? They are very much essential for the mathematical logical processes. Infinities and Infinitesmals are much required for scientific notations and derivations. As numbers cannot exist without zero, they cannot be operated effectively in higher mathematics and physics without infinity. Definitely, "Set theory is an accommodative method of using different infinities effectively in mathematics and "Renormalization" in physics.

Bibhutibhushan Datta & Avadesh Narayan Singh[14], Subhash Kak[15], Cart B. Boyer[16] and others have opined that the exact form of zero as used by the Hindus in the earliest time is doubtful, as they employed both dot and round symbols. Kak tried to interpret that the sign might be round or oval, but it may be noted that human being may or can write in any way or form according to his habit. We can say that it might also be in the form of ellipses of different axes! However, the truth is that Bindu is the simple way of representing Zero and round form (other elliptical forms are aberrations of human writing) for mathematical operations. The Bindu and Bindu written above a semicircle have been characteristically used in many inscriptions and coins to denote OM, the primordial word and sound of language. Such symbol is also found at the beginning of ancient texts (Palm leaves, scrolls etc.) In simpler forms definitely, they can be taken

as zero and infinite symbols written together, later theologized to denote Vinayaka, the first person to invent the art of writing and originator of every thing. In some places, the dot is placed inside a circle. Therefore, Zero as the origin of number system is supported by the Indian philosophical and mathematical texts.

Infinity is written as two Zeroes (characteristically a small zero touching a big zero) joined together horizontally ∞. Inscriptional evidences clearly prove that the symbol of infinity must have evolved only from India. Originally, higher numbers such as 1000, 6000, 10,000, 20, 000 etc were characteristically written with the combination of involved numbers slowly forming the symbol ∞. Such symbols ∞,∝, α,−∝, +∝, │−∝ etc. are found in Nanahat, Kushana, Ksatrapa and Andhra inscriptions and in the Ksatrapa coins (2 to 4 cent CE) and Jayagayapeta and Pallava grants (4 cent. CE) They are found in Indus Valley Civilization also (3500 BCE). Literary evidences and tradition give ample support to this fact. It is pertinent to mention that the name of the First Chief Serpent Adi Sesha with thousand hoods is Ananta and its coiled form is nothing but the infinity symbol. It is associated, compared and equated with Krishna (Bhagvat Gita X.28-29), Vishnu (forming the couch in the ocean of milk), churning of ocean leading to the creation, Balaram, 'Varaha' the third incarnation of Vishnu at the begining of Kalpa. Such coiled snake form is depicted very often in sculptures and erected on the banks of rivers, tanks and ocean and under trees.

As already mentioned that John Wallis (1616 - 1703) was the first to introduce the symbol ∞ for infinity in his works "Arithmetica infinitorium" and "De Sectionibus conics"[17]. It is claimed that he adopted the symbol taken from a Roman inscription dated to 36 CE, where it was used to denote 10,000. It was inscribed as two zeros joined together placed within a frame[18]. Initially there was opposition to his usage, as the symbol used had no consistency

with the concept and understanding of infinity in the western context. Voltaire criticized that "...the use of the love knot does not add to our understanding of the concept." However, Newton (1642 - 1727) and Gottfried Wilhelm Vo Leibniz encouraged its usage by the mathematicians. But about the infinity, the mathematicians were very much confused[19].

How John Wallis all of a sudden could have used the symbol ∞ for much avoided infinity (in 1655, 1657 or 1665)? Karl Menninger's suggestion of its Roman origin is not convincing, as Romans never had any zero symbol to represent higher numbers like 10,000 using zeros. The correspondence between famous scientists, astronomers and others like Newton, Johannes Kepler, Oldenberg, John Bentley, Josephy Priestley, Euler and their writings against Indian astrology, astronomy and chronology prove that they wanted to hide the source taken from India. Therefore, John Wallis must have taken the symbol from the Indian sources, as the symbol was used exactly in the same form with the evolution and representation. The inscriptional usage of it proves the consistency as pointed above[20]. Infact, the Jesuite and other missionairies were continuously collecting scientific, astronomical and mathematical manuscripts, tables, charts etc., from India and sending to them for further study and research[21].

7. The Opposition to Zero and Infinity by the West

It is intriguing as to why the westerners showed much opposition, hatred and horror to the concepts as well as the usage of zero and infinity. The opposition to zero has already been pointed out. The Infinity had been opposed by the west from the Greeks to modern scientists for reasons best known to them. The writers tactfully record that while the Greeks avoided infinity by their "elegant method of exhaustion", the modern mathematicians with saying that it was an illusion. D'Alembert looked upon Infinity as nothing but a limit which the finite

approaches without ever reaching it.

The Basis of Operation of Zero and Infinity

The etymological origin of words used for Zero and Infinity are shown to prove that they have been conceptually originated and evolved from the Indian sources. Bhaskaracharya gives six rules for the operations of Zero with other numbers:

1. If zero is added or subtracted from any number, that number remains as it is; its positivity or negativity remains the same. But if from zero something is removed, its sign changes.

2. If zero is added to (1)+3, (2)-3 or (3) zero, what are the respective sums?

3. And if from zero (1)+3, (2)-3 or (3) zero is subtracted what will be the remainder in each case?

4. If zero is multiplied by any number or divided by any number, the product is zero. If any number is divided by zero the resultant is a quantity with zero divisor (khahar).

5. Give the result when (1), zero is multiplied by 2, (2) zero is divided by 3, (3) 3 is divided by zero, (4) zero is squared. And give the square root of zero.

6. Just as at the time of deluge all beings enter the endles changeless and at the time of creation emerge from the Infinite God and by these acts the infinite remains unaffected, in the same way to this quantity with zero divisor if we add or from this we remove large qauntities, there cannot be any change in it.

Thus, the operations of zero and infinity with other numbers and as well as with themselves are given as follows:

$$n + 0 = 0 \qquad n + \infty = \infty$$
$$n - 0 = 0 \qquad n - \infty = \infty$$

$$n / 0 = \infty \qquad n / \infty = 0$$
$$n \times 0 = 0 \qquad n \times \infty = \infty$$

$$0 + 0 = 0 \qquad \infty + \infty = \infty \qquad 0 + \infty = \infty \qquad 0 / \infty = 0$$
$$0 - 0 = 0 \qquad \infty - \infty = \infty \qquad \infty + 0 = \infty \qquad \infty / 0 = \infty$$
$$0 / 0 = 0 \qquad \infty / \infty = \infty \qquad 0 - \infty = 0 \qquad 0 \times \infty = 0$$
$$0 \times 0 = 0 \qquad \infty \times \infty = \infty \qquad \infty - 0 = \infty \qquad \infty \times 0 = 0$$

The basis of operations is explained in the theological way and it can be called as Indian Mathematical Logic, like Modern Mathematical Logical dealing with the number theory etc. For example, the entire process can be explained simply by taking the definition and properties of ॐ:

"*ॐ. The invisible is the whole, the Visible is the whole. From the Whole, the Visible Universe has come out. The Whole remains ever itself even though the infinite Universe has come out of it*"

Now, let us substitute ॐ with 0,

"*0. The Invisible is the Whole (represented in sound form),From the Whole i.e, zero, the Visible numbers have come out. The Whole remains ever itself even though the infinity has come out of it*".

Who must have thought of infinity and its operations? Only, who has thought about very very small and very very great numbers. The ancient Indians have on record to prove that they knew numbers upto 10^{12} (Paradha- Yajurveda), 10^{22} (Taittiriya Upanishad), 10^{53} = tallaksana (Lalita Vistatara), 10^{87}(Valmiki Ramayana), 10^{140} = asankhyaye (Kacchyana's Pali Grammar).

In Chhandogya Upanishad, the conversation between Satyakama Jabala and his guru reveals interesting details about Infinity. Though it appears to be philosophical, the mathematical mind can easily understand the significance of it. The Fire instruct Jabala that one who meditates on endless (anantavan) becomes endless (anantavan). One

who knows that one foot of Brahman consisting of four parts, thus and meditates on it as the endless (anantavan), wins the endless regions (4.6.4.) That is at infinity the number looses its identity and the result is infinity. Thus, the above operations are clearly proved.

Again the Fires (the Garhapatya, Anvarharypacana and Ahavanya) tell him that Prana (life) is Brahman, ka (joy) is Brahman, kha (ether/Infinity) is Brahman (4.10.4) Jabala characteristically replies that he understands that Prana is Brahman, but does not understand Ka and Kha. The Fires clarify that Prana (life) and Akasa within the heart are related to it (4.10.5). That is, the pleasure centered in Akasa, not the ordinary pleasure, and the Akasa as the centre of happiness not the physical Akasa, are to be meditated upon. Kha qualifying Ka limits "pleasure" to unworldly ones; and ka limits Akasha to non- physical. So by thus making each word would qualify the other. This can be put in the mathematical form as follows:

$$\int_{\text{ख}}^{\text{क}} (\text{life}) \text{ Brahman} = \text{Pleasure to unworldly ones}$$

when Life is integrated with respect to Brahman from the limits Kha (Infinity) to Ka (Absolute unity) it becomes ka.

$$\int_{\text{क}}^{\text{ख}} (\text{Life}) \text{ Brahman} = \text{Akasa to the non- physical}$$

when Life is integrated with respect to Brahman from limits Ka (Absolute unity) to Kha (Infinity), it becomes Infinity.

Thus, Ka (Absolute Unity) =Kha (Infinity) and   Kha (Infinity) = Ka (Absolute Unity).

Again to show how the Prana is the greatest to be obtained, it is explained (Chapter VII) that-

Prana (Life) > Aspiration (asha)> Memory (smar)> Ether (Akash)> Fire (tej) > Water (Ap) >

Food(annam)>Strength (balam)> Understanding (vigyanam) > Contemplation (dhyanam) > Intelligence (chittam) > Will (sankalpa) > Mind (man) > Speech (vak) > Name (nam).

Here, the concept of relativity has been applied. It can be noted in explaining the big bigger and biggest of all.

Infinitesimal quantities are also explained in different categories as follows:

Honey is collected from different flowers of plants and trees, but the taste is same (6.9.1-2) Ocean contain waters of different rivers, but the colour is the same (6.10.1-2). The water sucked by a tree percolates to all parts for the growth (6.11.1-2). Bunyan tree grows very big. It contains small fruits. If a fruit is broken, we get small seeeds. If a seed is broken, nothing is seen because, it is seed itself (612.1-2). If salt is mixed with water it is dissolved and disappears, but its presence can be understood with taste (6.13.1-2). Thus to understand diminutive / microscopic particle sizes, the relative concept is applied in comparison.

Brahadaranyaka Upanishad typically brings out the concept of Infinity specifically. It is mentioned that the Almighty manifested in 10 incarnations. Then, many innumerable manifestations followed. He is tens and thousands and Infinity (*dasa cha shasrani bahuni cha anatani* (2.5.19). Here, not only the tens and thousands are related to infinity, but the word Ananta is used to denote infinity.

The following conversation specifically discussess about the nature of Infinity: The King says that one should worship God as "Ananta - All pervasive or Infinite". Yajnavalkya asks, "What is the nature of Infinity?". He replies, " The endless quarters... to whatever direction one may go, one never gets at its end..... It is Infinity" (4.1.5). Again, Infinity is succinctly explained (4.3.33) by comparing the Happiness (ananda) of man with that of Manes,

Gandharva etc., mathematically as follows:

Man's happiness= Mane's Happiness / 100
 = Gandharv's Happiness / 1000
 = God's Hapiness / 1000000
 = God's (by birth) Happiness / 100000000
 = Prajapati Brahma's Happiness / 10000000000
 = Brahma's Happiness / 100000000000000

Therefore, Brahma's Happiness = Human's Happiness x 10^{12}. When it is expressed in reverse, we can write that Human's Happiness is 10^{-12} of that of Brahman. Similarly, the human years with that of Brahman are compared:

1 Brahman day time = 4,32,00,00,000 human years
1 Brahman night time = 4,32,00,00,000 human years
Therefore, 1 Brahman day & night= 8,64,00,00,000 human years
From this, the Brahma's age = 1.5768 x 10^{14} human years
or 1 Human year = 0.6342 x 10^{-14} .

Here, the important point is how the infinitesimals are compared and figuratively explained. Thus, Ananda (hapiness) becomes Ananta (Infinity). Though ananda is immeasurable, it is quantified for comparison, then, Brahma's Happinesss is quantified in comparison with man,though Brahma cannot have any hapines. Infact, Anantananda(Infinite bliss) is mentioned in the context of divinity. Therefore, by giving such quantification Infinity is clearly implied. Similarly the equations can be put in reverse saying that Mane's happiness is 100 times that of man and so on. This proves the ability of scriptures conveying the mathematical concepts relatively.

Brahma Sutras use the word "bhuma" to denote infinity and itis equated with Brahman (1.3.8). Individual and soul become one with the Infinite (3.2.26). Though iva and Brahma appear to be different they ate connected together. It is explained in this way that " on account of both (i.e., difference and non-difference) being taught (by the (Sruti) the relation of Jiva and Brahman is to be taken like that between a serpent and it's coil" (3.2.27).

First, Jiva (zero) is equated with Brahman (the Absolute) and then, it is said that they are equal in Infinity ($0 + 1= 1$ and $0 + \infty = \infty$ & $1 + \infty = \infty$). Then, the relation between Jiva and Brahman is compared with the serpent (head resembles zero) and it's coil (resemble infinity). Here, perhaps the relation between zero and infinity is brought out clearly and incidentally, the symbol of infinity i.e. the coil of a serpent mentioned can also be noted).

8. Conclusion

The conceptualisation of Zero and Infinity is connected with deep epistemologization of mathematical processess. Both cannot be separated. As the ancient Indians used such combined methodology consistently with their scriptures, related rituals and connected constructional activities, the origin of zero and Infinity in India is well authentic supported by literary, archeological, epigraphical and other evidences. Many times, the results have been condensed and mythologized or compared with the then popular incidences for easy rememberance. As pointed out above in the case of six rules for working with zeroes, books can be written for each rule. But, absence of such worked out examples cannot be a reason to deny the existance of such knowledge during the material period or even before, as nothing comes from vacuum. Western and Arab writings themselves clearly prove that Indian mathematical knowledge went from India to their respective destinations. Therefore, the available texts should be completely interpreted in the modern context acceptable to the concerned scholars for appreciation and recognition.

Explanation of Abbreviations

1. BCE = before Christian Era
2. CE = Christian Era

3.   Cent. = Century

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14. Ibid, p.244-45.

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16. V.S. Apte, English Sanskrit Dictionary, Motilal Banarsidas, New Delhi, 1964.

17. McGraw Hill Encyclopedia of science and Technology (5th Ed), USA, 1987, Vol.14, p. 796. In the acceptance of zero as a number, the orient is certainly ahead of the occident.

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22. Karl Menninger, opt. cit, p.245.

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24. George de Buffon (1707-1788) rejected his infinity as an illusion, seeing it in infinitely large and small nothing but the idea of a quantity increasing or decreasing wihtout limit.

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